Math 210- Exam 1

Fall 2009

1. If $S \subset \mathbb{R}$ is a nonempty bounded set, and $I_S = [infS, supS]$; show that $S \subset I_S$: Moreover, if J is any closed bounded interval containing S; show that $I_S \subset J$.

2. Let $S \subset \mathbb{R}$. show that if $u = supS$ then for every integer n, the number $u - \frac{1}{n}$ is not an upper bound of S, but the number $u + \frac{1}{n}$ is an upper bound of S.

3. Let $X = (x_n)$ be a sequence of positive real numbers such that $lim_{n\to\infty}(x_{n+1}/x_n) = L > 1$. Show that X is not bounded and hence X is not convergent.

4. Let $X = (x_n)$ be a sequence of real numbers such that X does not converge to $x \in \mathbb{R}$. Show that for some ϵ_0 there exist a subsequence (x_{n_k}) such that $\mid x_{n_k} - x \mid \geq \epsilon_0$ for all $k \in \mathbb{N}$.

5. If $\sum a_n$ with $a_n > 0$ is convergent, and if $b_n = (a_1 + a_2 + ... + a_n)/n$ for $n \in \mathbb{N}$, then show that $\sum b_n$ is always divergent.
   Hint: Show that $b_k \geq a_1/k$ and then look at the partial sums of $\sum b_k$.

6. Let g be a rational function (ratio of two polynomials). Show that $\sum g(n)r^n$ converges absolutely for $\mid r \mid < 1$ or diverges if $\mid r \mid > 1$. Discuss the possibilities if $\mid r \mid = 1$.